

Two-Sample t-test Dependent Samples

Course: Statistics 1

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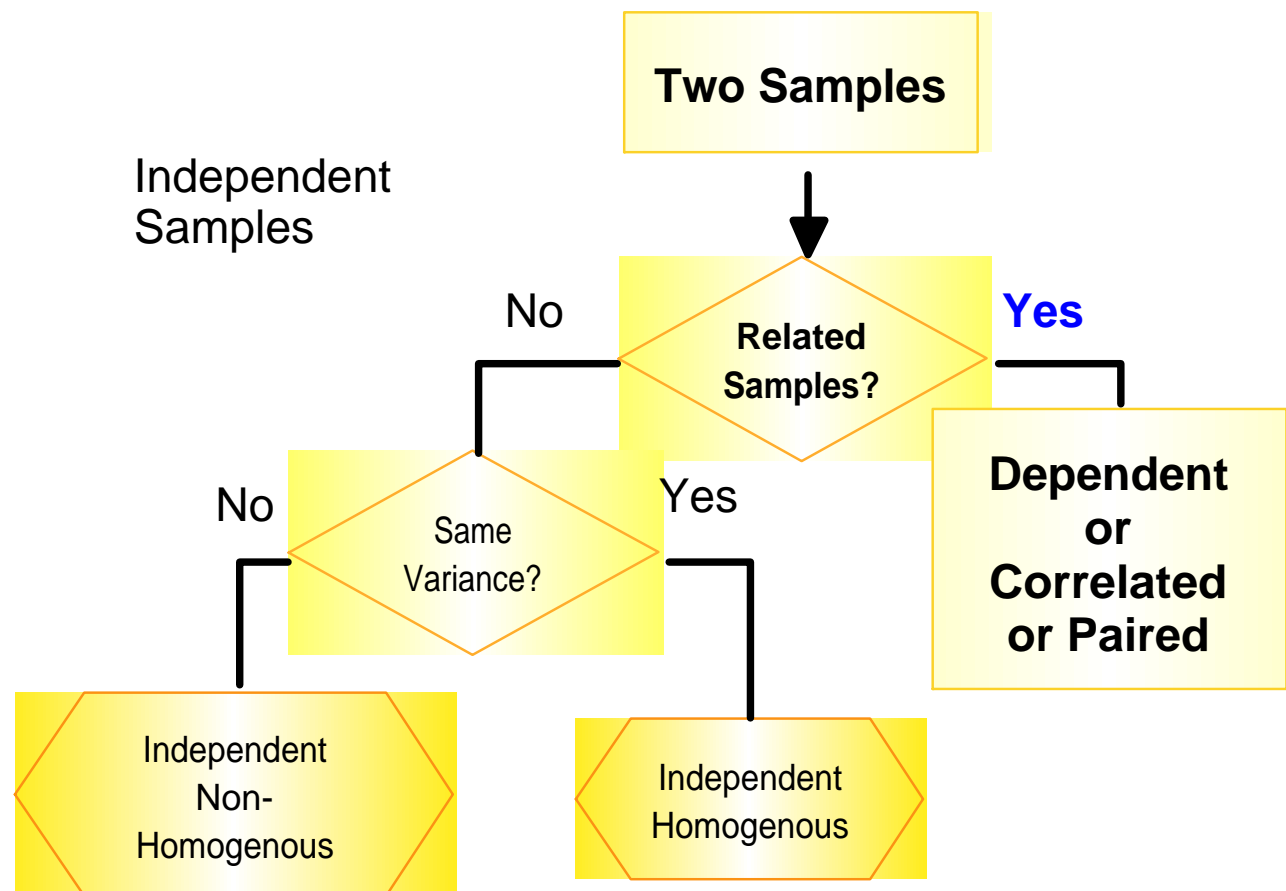
Example

- Examples:
 - Pretest-Posttest studies
 - Husband/wives studies
 - Twin studies
- Case:
 - Dependent Samples: Correlated Samples
 - Sample: Cars' MPG with and without additives

Sample

Car	With Additive	Without Additive	d
1	35	38	-3
2	40	39	1
3	42	40	2
4	34	40	-6
5	46	46	0
6	32	37	-5
7	32	36	-4
8	40	38	2
9	45	43	2
10	46	47	-1
Mean	39.2	40.4	-1.2
SD	5.613476	3.747592	$S_d = 3.08$

Dependent Samples



Step 1: Hypotheses

- Null, H_0 (*difference in means is 0*)

$$\mu_1 - \mu_2 = 0$$

- Alternative, H_a (Non-Directional)

$$\mu_1 - \mu_2 \neq 0$$

Step 2: Set Rejection Criterion

- Significance Level: $\alpha = 0.05$
- Critical value (t -distribution, $df = n - 1 = 9$)
 - Two-tailed (non-directional)
 - $t_{cv} = 2.262$
 - Reject H_0 if $t_{stat} \geq 2.262$

Step 3: Compute Test Statistics

$$\text{Std Error: } s_{M_d} = \frac{s_d}{\sqrt{n}} = \frac{3.08}{\sqrt{10}} = 0.98$$

$$\text{Test statistics, } t = \frac{M_d - 0}{s_{M_d}} = \frac{-1.2}{0.98} = -1.22$$

Step 4: Confidence Interval

- CI = Statistics +/- Critical Value (Standard Error)
- Mean of Difference, $M_D = -1.2$
- $t_{cv} = 2.262$ (two-tailed, $df = 9$ and $\alpha = 0.05$)
- $CI_{95} = -1.2 \pm 2.262(0.98) = -3.42$ to 1.02

Step 5: Effect Size

- ES = Mean Difference / Standard deviation = $(M_1 - M_2)/s$
- Previously, $S_d = 3.08$ (Standard deviation of difference)
- ES = $(-1.2 - 0)/3.08 = -0.39$; So absolute $d = 0.39$
- **Conclusion:** Small effect
 - (between 0.2 and 0.6: Small to Medium)

Step 6: Decision

- **Do not** reject H_0 :
 - 1. $t_{stat} < t_{cv}$ or $1.23 < 2.262$
 - 2. Hypothesized population mean difference of 0 is within CI_{95}
 - CI_{95} : -3.42 to 1.02
 - 3. ES = 0.38 is small to medium
- **Conclusion:** There is no reason to believe that cars with additives performance (by MPG) is significantly different from cars without additives

SPSS Outputs 1

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Words	39.2000	10	5.61348	1.77514
	No_Additives	40.4000	10	3.74759	1.18509

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Words & No_Additives	10	.857	.002

Correlation: Both samples are correlated, since p-value = **0.002** < **0.05**

SPSS Output 2

Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pair 1	-1.20000	3.08401	.97525	-3.40617	1.00617	-1.230	9	.250

t-Test: Do not reject null hypothesis – mean of difference is 0, since:

1. $t_{test} = 1.23 < t_{cv} = 2.262$ (two-tailed, $df = 9$, $\alpha = 0.05$)

2. $p\text{-value} = 0.250 > 0.05$ and

3. CI_{95} do contain 0