Standard Score (z-score)

Course: Statistics 1

Lecturer: Dr. Courtney Pindling

Standard Deviation and Normal Curve

- For normal curve, the number of standard deviations about the mean gives the percent of data about the mean

- The standard normal curve has *mean = 0* and *standard deviation = 1*

- Normal Curve:
  - 68% of data within -1*S* and +1*S* of mean
  - 95% of data within -2*S* and + 2*S* of mean
  - 99% of data within -3*S* and + 3*S* of mean

Standard Normal Curve
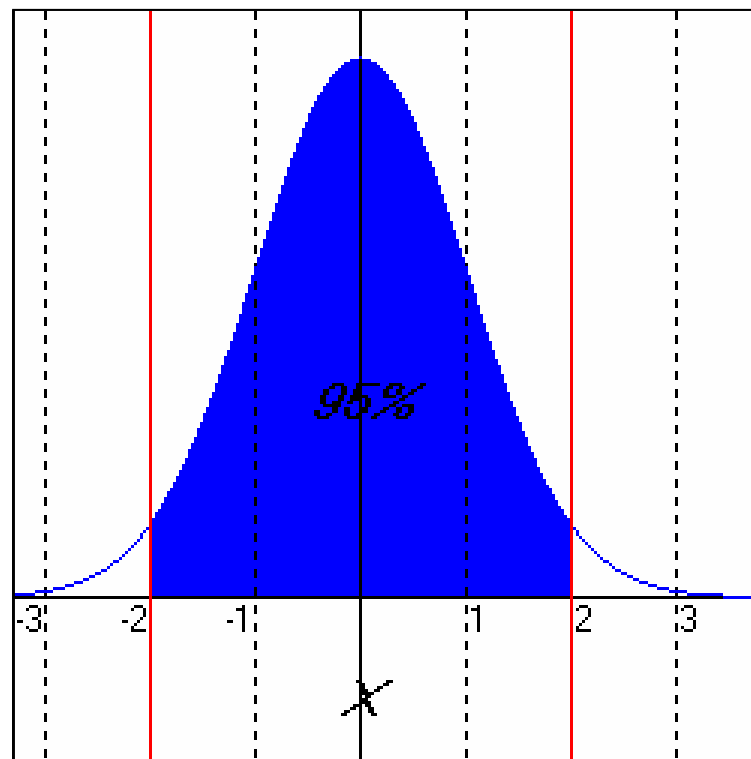
Standard Score

- **_z-score_**, standard score, is the number of standard deviations from the mean

- Formula for **z** is:

$$z - score: z = \frac{\text{Observed Score - Mean}}{\text{Standard Deviation}} = \frac{X - M}{SD}$$

  - _z_ is the z-score
  - _X_ is the observed score
  - _M_ is the mean
  - _S_ is the standard deviation of the data set

Use of z-score
Standard Score

- A simply way of tell how far a score is from the mean in standard deviation units
  - *Positive z score indicates score is above the mean*
  - *Negative z score indicates score is below the mean*
- To find the location of a score relative to the normally distributed variable
- It is a normalized measure that allows for comparisons between different distributions
- It is a derived score (from original data scores)

Standard Scores of Data

$$z = \frac{X-M}{SD} = \frac{82-85.7}{2.50} = -1.48$$

$$z = \frac{X-M}{SD} = \frac{89-85.7}{2.50} = +1.32$$

| **82** | 83 | 84 | 85 | 85 | 86 | 86 | 87 | **89** | 90 |

Comparing Different Distributions

Course	Raw Data	Mean	Standard Deviation
Math	78	75	6
Science	115	103	14
English	57	52	4

Which Subject the John did best in relative to rest of class?

Comparing Different Distributions

$$\text{Math: } z = \frac{X - M}{SD} = \frac{78 - 75}{6} = +0.5$$

$$\text{Science: } z = \frac{X - M}{SD} = \frac{115 - 103}{14} = +0.86$$

$$\text{English: } z = \frac{X - M}{SD} = \frac{57 - 52}{4} = +1.25$$

John did best **English** relative to rest of class, since his **z-score was highest** in English.

Other Standard Scores

- Disadvantage of *z*-scores
    - *Negative values, mean = 0, and decimal fractions*
- Alternate to *z*-score are such scores as *T* scores
- *T* score calculated by formula (*M = 50, SD = 10*)

$$t \text{ scores: } t = 10z + 50$$

- Next slides show other standard scores calculated by formula:

$$\text{Other Standard Scores: } \sigma z + \mu$$

T score Example

$$z = \frac{X - M}{SD} = \frac{82 - 85.7}{2.50} = -1.48$$

$$t \text{ scores: } t = 10z + 50$$

T = 10(-1.48) + 50 = **35.2**

82	83	84	85	85	86	86	87	89	90

Standard Score Systems

Other Standard Scores: $\sigma z + \mu$

System	μ	σ
z scores	0	1
T scores	50	10
GATB	100	20
CEEB	500	100
IQ	100	15

GATB – General Aptitude Test Battery
CEED – College Entrance Examination Board
IQ – Intelligence Quotients

Normal Curve and Standard Scores

z-scores	-3	-2	-1	0	1	2	3
T scores			40	50	60		
GATB			80	100	120		
CEED			400	500	600		
IQ			85	100	115		