One-Sample T Test

Unknown Population Variance

Course: Statistics 1

Lecturer: Dr. Courtney Pindling

Hypothesis

Null hypothesis, $\boldsymbol{H_0}$**:**

sample mean is the same as the population mean

$\boldsymbol{H_0}$: *sample mean =* μ

Alternative Hypothesis, $\boldsymbol{H_a}$**:**

a. *sample mean is not the same as population mean*, *or*

b. *sample mean is > population mean, or*

c. *sample mean is < population mean*

Assumptions

- We know:
  - Given population mean
  - Unknown population variance ($\mathbf{s}^2$)
  - Use sample variance, $\mathbf{s}^2$ to estimate $\mathbf{s}^2$
    - Standard error is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Test Statistics

- Since we do not know the population variance, we use the ***t-distribution*** rather than the standard normal distribution, z:

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

- At large sample size, t-distribution approximates the normal distribution
- Robust enough to be used even for cases when we would use the z statistics

Example 1

- We need to test the hypothesis that a sample of **n = 25,** with a mean of **170** and standard deviation of **6** is the same as the population mean of **165** at a significance level of alpha

  = **0.05** (two-tailed test) whether:
  - ***H_0***: *170 (**M**) = 165 (**μ**)*
  - ***H_a***: *M ≠ μ*
  - *Let a = 0.05*

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{6}{\sqrt{25}} = 1.2$$

$$t_{stat} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{170 - 165}{1.2} = 4.17$$

Conclusion: Example 1

- Two-Tailed Test (*non-directional hypothesis*)
- Probability p-value (reject H_0):

  Pr**(t >= 4.17) = **0.000171,

  So p-value = 0.000343 =< 0.05

  MS Excel: TDIST (test statistics, df, tails)

  MS Excel: TDIST(4.17, 24, 2) = 0.000343 = p-value

- Critical Value (reject H_0):
  - t_{cv} = *2.064* (*df* = 24, a = 0.05)

  t-statistics or *t*-stat >= $t_{a/2}$ ***or*** *4.17 >= 2.064*

  So sample mean of 170 is not equal to population mean of 165

Example 2

- A study was done to evaluate whether the population mean was **greater** than 7. A sample of **60** yield ***M*** = **7.25** and ***SD*** = **1.05.**
  - ***H_0***: *7.25 (**M**) = 7 (**μ**)*
  - ***H_a***: *M **>** μ*
  - *Let a = 0.05*

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{1.05}{\sqrt{60}} = 0.1356$$

$$t_{stat} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{7.25 - 7}{0.1356} = 1.84$$

Conclusion: Example 2

- One-Tailed Test (*directional hypothesis*)
- Probability p-value (reject H_0):

  Pr*(t >= 1.84) =* ***0.0354,*** *So p-value = 0.0354 =< 0.05*

  MS Excel: TDIST (test statistics, df, tails)

  MS Excel: TDIST(1.84, 59, 1) = 0.0354 = p-value

- Critical Value (reject H_0):

  $t_{.cv}$ = *1.671* (*df* = 59, a = 0.05)

  t-statistics or *t*-stat >= t_a ***or*** *1.84 >= 1.671*

  So sample mean of 7.25 is not equal to population mean of 7