One-Sample Mean

Know Population Variance

Course: Statistics 1

Lecturer: Dr. Courtney Pindling

Hypothesis

Null hypothesis, $\boldsymbol{H_0}$**:**

sample mean is the same as the population mean

$\boldsymbol{H_0}$: *sample mean* = μ

Alternative Hypothesis, $\boldsymbol{H_a}$**:**

a. *sample mean is not the same as population mean, or*

b. *sample mean is > population mean, or*

c. *sample mean is < population mean*

Assumptions

- We know:
  - Population mean
  - Population Variance ($\mathbf{s}^2$)

Standard Error

- Repeated sampling will result in a normal distribution of sample mean
- Average of these means will be close to ***μ***
- **Use one sample to estimate**
- The ***standard error of sample mean*** is the standard deviation of the sampling distribution of the sample mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

N	Mean, *M*
120	35.6
240	40.1
500	33.5
400	34.1
250	38.5
450	36.4
…	…

Test Statistics

- Since we know the population variance, we use the standard normal distribution, Z:

$$z = \frac{\text{Sample Mean - Population Mean}}{\text{Standard Error}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Example 1

- A company produced golf balls with a driving distance of **295** yards, **s** is assumed to be **12** yards. A sampling of **50** golf balls shows the mean driving distance of **297.6** yards. Is the mean 295 yards, based on this sample?
  - H_0: $\mu = 295$
  - H_a: $\mu \neq 295$
  - *Let a = 0.05*

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = 1.7$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{297.5 - 295}{1.7} = 1.53$$

Conclusion: *p*-value

- Two-Tailed Test (*non-directional hypothesis*)
- Probability p-value

  z =< -1.53   and z >= 1.53

  Pr*(z =< -1.53) =* ***0.063*** *and* ***Pr****(z >= 1.53) =* ***0.063***

  So p-value = ***2****(0.063) =* ***0.126***

- No **<u>not</u>** reject H_0:

  p-value (0.126) > 0.05

  So no need to adjust company's golf making process

Conclusion: *Critical Value*

- Two-Tailed Test: a = 0.05 and area of both tails beyond critical value is a/2 = 0.025
- Critical region (do not reject H_0):

  $z =< -1.96$ *and* $z >= 1.96$

  $-z_{0.025} = -1.96$ *and* $z_{0.025} = 1.96$
- No **not** reject H_0:

  $\boldsymbol{z = 1.53}$ is within – 1.96 and 1.96

Conclusion: *Confidence Interval*

- Two-Tailed Test: a = 0.05, so a/2 = 0.025

  Critical value is $z_{0.025} = 1.96$

  95% Confidence Interval:

  mean +/- 1.96 (Std error)

$$\bar{x} \pm z_{0.025} \frac{\sigma}{\sqrt{n}}$$

$$297.6 \pm 1.96 \frac{12}{\sqrt{50}}$$

- Confidence Interval (do not reject H_0):

  Outside: ***294.3 to 300.9***

$$297.6 \pm 3.3 = 297.6 + 3.3 \text{ and } 297.6 - 3.3$$

- No **not** reject H_0:

  Since $\boldsymbol{\mu = 295}$ is within 95% CI

294.3 to 300.9

Example 2

- A company produced golf balls with a driving distance of **295** yards, **s** is assumed to be **12** yards. A sampling of **50** golf balls shows the mean driving distance of **297.6** yards. Is the sample mean **higher** than 295 yards?
  - H_0: $\mu = 295$
  - H_a: $\mu > 295$
  - *Let a = 0.05*

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = 1.7$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{297.5 - 295}{1.7} = 1.53$$

Conclusion: One-Tailed Test

- One-Tailed Test (*directional hypothesis*)
- Probability p-value (do not reject H_0):

  Pr*(z >= 1.53) =* ***0.063,*** *So p-value = 0.063 > 0.05*
- Critical region (do not reject H_0):

  $z_{0.95}$ *= 1.64 (upper tail, a = 0.05), So z-test (**1.53**) < 1.64*

- So no need to adjust company's golf making process