

The Normal Distribution

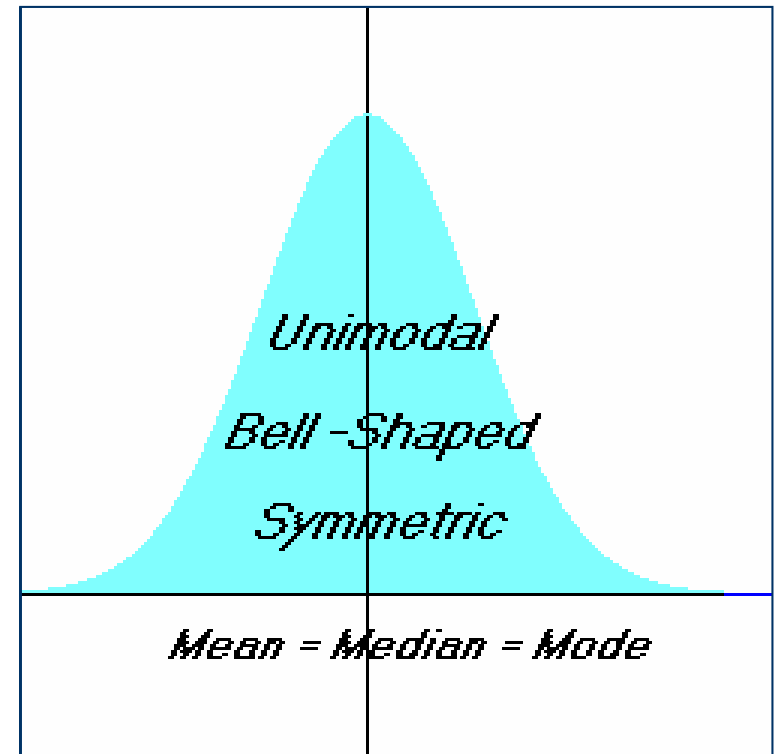
Course: Statistics 1

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The Normal Distribution

- **Family** of Normal Distributions
 - *Mean and Standard Deviations*
- **Symmetrical, bell-shaped, and unimodal** distribution
- Represents distributions of **continuous** variables
- An assumption of many inferential statistical methods



Standard Score

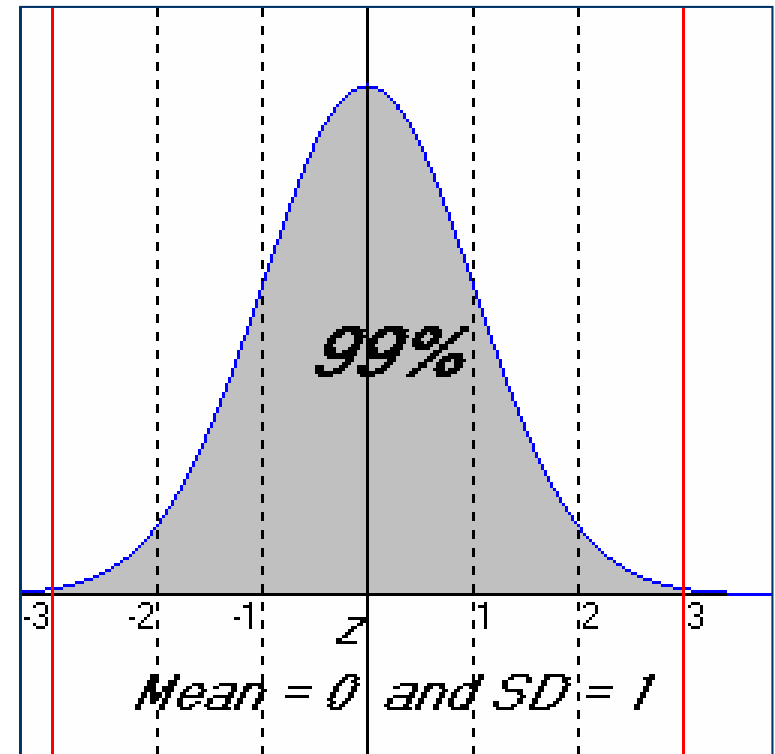
- **z-score**, standard score, is the number of standard deviations from the mean
- Formula for **z** is:

$$z - score: z = \frac{\text{Observed Score} - \text{Mean}}{\text{Standard Deviation}} = \frac{X - M}{SD}$$

- z is the z-score
- X is the observed score
- M is the mean
- S is the standard deviation of the data set

The Standard Normal Distribution

- The Standard Normal Distribution is the distribution of **z-scores**
- Area under curve is 1 or 100%
- Cumulative percent z
 - % area from left to z-value
- Area between z-scores:
 - 68% between $z = -1$ and $z = +1$
 - 95% between $z = -2$ and $z = +2$
 - 99% between $z = -3$ and $z = +3$



Normal Curve and z-score

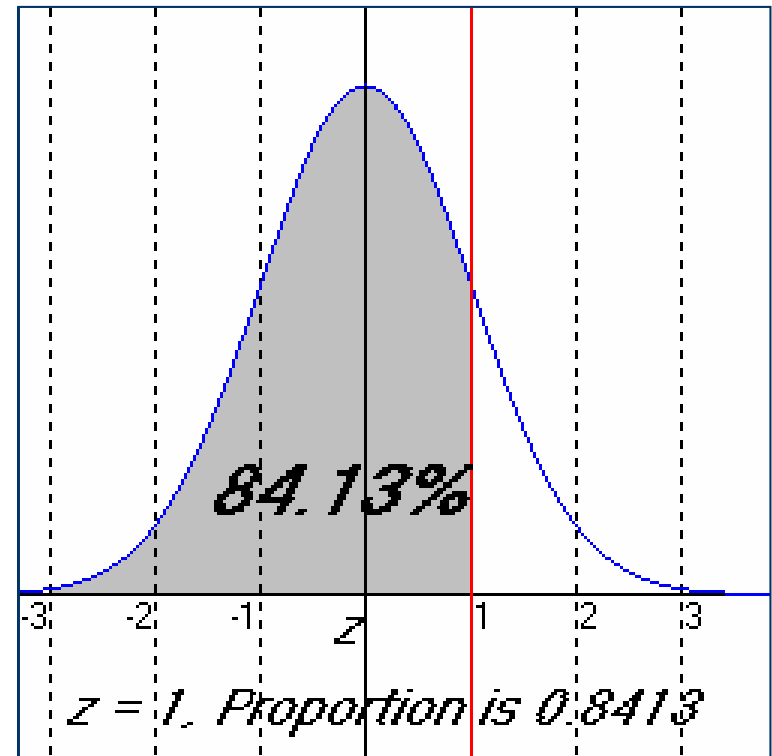
- Proportion (Percentage) Under Curve
 - Between pairs of z-scores
 - Less than specified z-score
 - Greater than specified z-score
- Percentile, $P_{\text{percent value}}$
 - The raw score that a given % of distribution is less than or equal to
- Percentile Rank, PR_x
 - The percentile of a given raw score
- Normal Curve Equivalent (NCE)
 - A normalized standard score; **$NCE = 21Z + 50$**

Cumulative z-score Table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525

Percent when $z = 1$

- From cumulative z table
- $Z = 1$ yields 0.8413
- Convert to Percent by multiplying by 100
- **84.13%** when $z = 1$
- So 84.13% of scores are $\leq z = 1$



Area Under Curve

Cumulative z-score Table

- Proportion or Percentage of z-score
 - Example 1: Percent $< z = 0$
 - Table $z = 0$ is 0.5, so **50%**
 - Example 2: Percent $< z = 1$
 - Table $z = 1$ is 0.8413, so **84.13%**
 - Example 3: Percent $< z = 1.65$
 - Table $z = 1.65$ is 0.9505, so **95.05%**
 - Example 4: Percent $> z = 1$
 - Since $84.13\% < z = 1$, so **15.87%** ($100 - 84.13$)
 - Example 5: Percent between $z = 1$ and $z = 1.65$
 - $95.05 - 84.13 =$ **10.92%**

Standard Scores Examples

$$z = \frac{X - M}{SD} = \frac{82 - 85.7}{2.50} = -1.48$$

$$z = \frac{X - M}{SD} = \frac{89 - 85.7}{2.50} = +1.32$$

Percent Rank, PR_x

Cumulative z-score Table

- The percentile of a given raw score
- Example 6: Percent Rank of $X = 82$
 - Calculated z-score is -1.48
 - Table $z = -1.48$ is 0.0694, so $\% < z = -1.48 = 6.94\%$
 - PR_{82} is **6.94%**
- Example 7: Percent Rank of $X = 89$
 - Calculated z-score is +1.32
 - Table $z = 1.32$ is 0.9066, so $\% < z = 1.32 = 90.66\%$
 - PR_{89} is **90.66%**

Z-score of a Percentage

- What value of z corresponds to ($<$ %):
 - 45%? Find 0.45 proportion in cumulative table and locate its z -score
 - Its z -score is between $z = -0.12$ and -0.13
 - So the 45% point represents a z score of about ***-0.125***

-0.2000	0.4207	0.4168	0.4129	0.4090	0.4052
-0.1000	0.4602	0.4562	0.4522	0.4483	0.4443
z	0	0.01	0.02	0.03	0.04

Percentile, $P_{\%}$

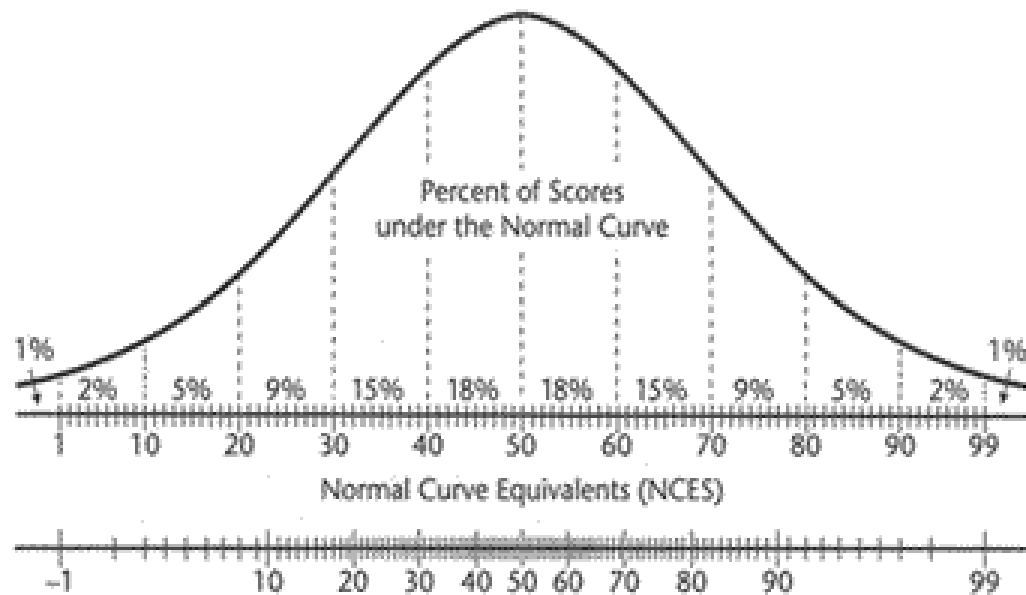
Cumulative z-score Table

- The raw score that a given % of distribution is less than or equal to $X = SD(Z) + M$ (where $SD = 2.5$, and $M = 85.7$)
- Example 8: P_{45}
 - Locate corresponding z-value for 0.45
 - Table shows $z = -0.125$ when proportion is 0.45
 - P_{45} is **85.39** since $2.5(-0.125) + 85.7$
- Example 9: P_{75}
 - Locate corresponding z-value for 0.75
 - Table shows $z = 0.675$ when proportion is 0.75
 - P_{75} is **87.39** since $2.5(0.675) + 85.7$

Normal Curve Equivalent - NCE

- A way of measuring where a student score falls along the normal curve
- NCE scores run from 1 to 99
- Standard Score: $M = 50, SD = 21.06$
- Similar to Percent Rank (1 to 100)
- NCE can be averaged
- Good for measuring school-wide gains and losses in student achievement
- $NCE = 21Z + 50$

NCE Comparisons



Useful Link

- Compute proportion area under curve
- http://davidmlane.com/hyperstat/z_table.html