Hypothesis Testing

Course: Statistics 1

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Introduction

Hypothesis testing can be used to determine whether a statement about the value of a population should or should not be rejected

It begins by making a tentative assumption about a population parameter

- Tentative assumption called ***null hypothesis***, H_0
- Opposite of H_0 is called ***alternative hypothesis***, H_a
- Uses data from sample to test two competing statements

Null Hypothesis

- Event, or, apparent effect, or difference is due to chance – purely random or by chance
- Typically the opposite of the researcher's hypothesis
- The opposite of the alternative hypothesis
- Rejecting the H_0 favors the H_a
- Null hypothesis:
  - H_0
  - *Event(1)* **=** *Event(2) or Event(1) – Event(2) =* ***0***

Alternative Hypothesis

- Event, or, apparent effect, or difference is **not** due to chance – significant
- The researcher's hypothesis
- The opposite of the null hypothesis
- Rejecting the H_0 favors the H_a
- Alternative hypothesis:
  - *Ha*
  - *Event(1) ≠ Event(2) or Event(1) >= Event(2)*

Significance

- Probability of an outcome given the null hypothesis, ***p-value***
- Low probability value indicates rejection of the null hypothesis
- Typically: reject H_0 if ***p-value =< 0.05*** or ***0.01***
- ***Alpha*** *or* ***a*** *or* ***a****: probability below which* H_0 *is rejected*
- *The* ***significance level*** *is the same as alpha,* ***a***
- *Rejection of* H_0 *at* ***a*** *means a statistical significance*
- *Statistically significant means the effect is not due to chance*

Type I and II Errors

- *Either **H_0** or **H_a** is true, but not both*
- *Hypothesis could lead to accepting **H_0** when it is false*
- *Type I Error: Rejecting the **H_0** when it is true (**a**)*
- *Type II Error: Accepting the **H_0** when it is false (**ß**)*

		Population Condition	
		H_0 True	**H_a True**
Condition	**Accept H_0**	Correct	Type II
	Reject H_0	Type I	Correct

Definitions I

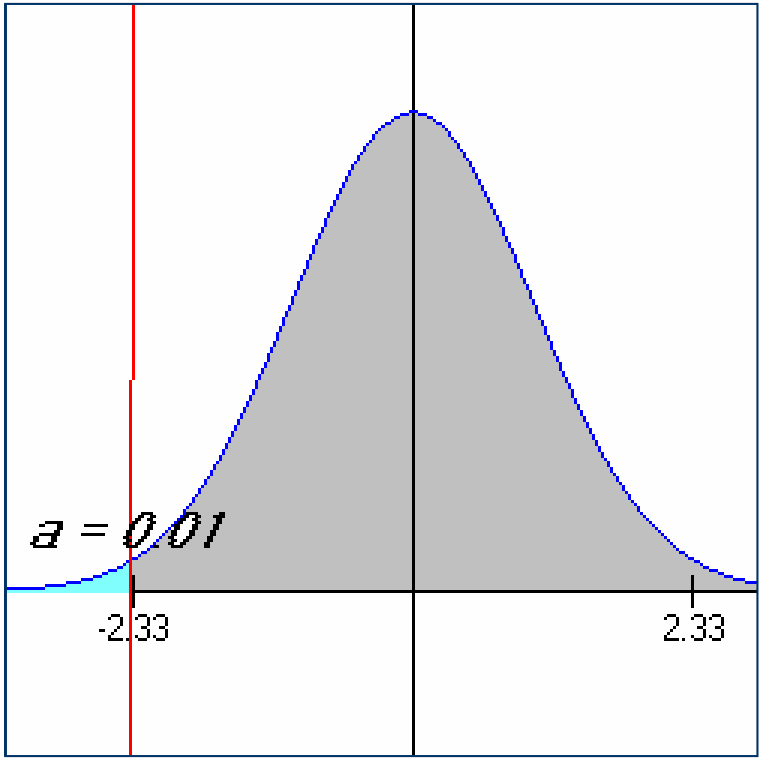
- ***Test Statistics***: computed statistics
  - (sample mean – reference) / standard error
  - Standard error = standard deviation / sqrt(***N***)
- ***p-value***: a probability that measures the support (of lack of) provided by the sample for the null hypothesis
- ***Critical value***: a number based on ***a***
  - 1.960 for ***a*** = 0.05
  - 2.576 for ***a*** = 0.01

Definitions II

- ***Confidence Interval***: computed statistics
  - mean +/- (Critical value)(Standard error) or
  - (mean difference) +/- (Critical value)(Standard error)

One-Tailed Test

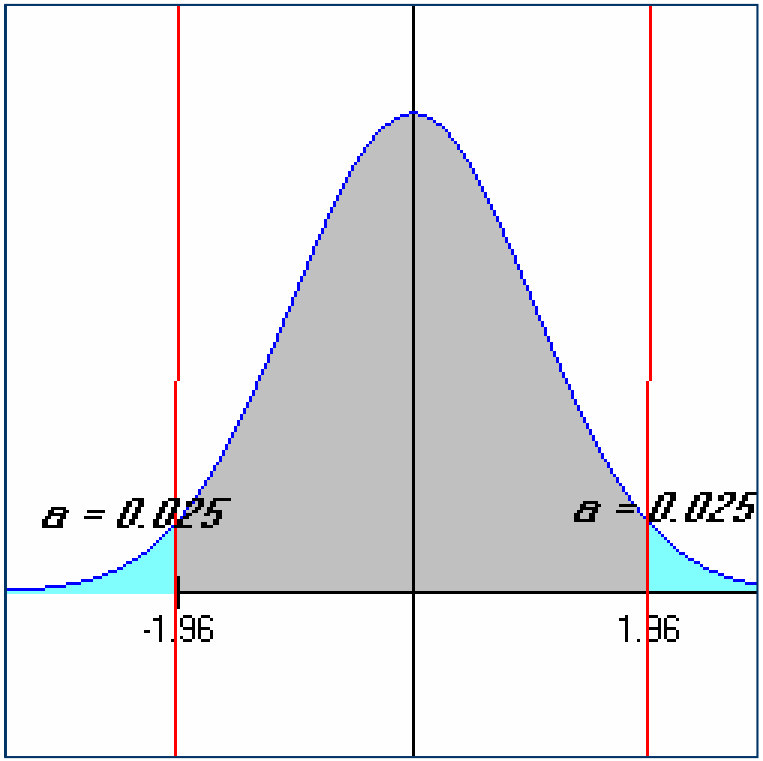
- *Hypothesis is **Directional***
- *Lower Tail Test*
  - H_0: $mean_1 = mean_2$
  - H_a: $mean_1 < mean_2$
- *Upper Tail Test*
  - H_0: $mean_1 = mean_2$
  - H_a: $mean_1 > mean_2$
- ***Reject H_0** if*
  - ***p-value =< a** or*
  - ***z =< -2.33 (a = 0.01)** or*
  - ***z =< -1.64 (a = 0.05)***



Two-Tailed Test

- *Hypothesis is **non-directional***
- *Two-Tailed Test*
  - H_0: $mean_1 = mean_2$
  - H_a: $mean_1 \neq mean_2$
- ***Reject H_0** if*
  - ***2(p-value) =< a** or*
  - ***z =< -1.96** or **z >= 1.96***

  (a = 0.05)



Steps of Hypothesis Testing

- **Step 1**. Develop the H_0 and H_a
- **Step 2**. Specify level of significance, ***a***
- **Step 3**. Compute test statistics from sample data
- **Step 4**. Obtain or compute ***p*-value** from Step 3
- **Step 4**. Reject H_0 if *p*-value =< **a**

Reject Null Hypothesis

- *Test Statistics* >= *Test Table Value**
- *p-value* =< *a*
- Confidence Interval does not contain “zero” (for mean difference) or
- Reference mean is outside the sample Confidence Interval (given ***a***)