One-Sample Correlation
Case II

Course: Statistics 1

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Introduction

Is the correlation coefficient significantly different from ***0*** or some reference value, *a*?

Test whether the linear relationship between x and y is significant by testing hypothesis about the population correlation coefficient, ρ_{xy}:

Case 1: H_0: $\rho_{xy} = 0$ ***or***

Case 2: H_0: $\rho_{xy} = a$

Note: Will only examine Case 2 in this lecture

Critical Value: (H_0: $\rho = a$)

Critical Value from standard normal distribution , *z* score

Fisher Z transformation

(change scale from ***r*** to *Z*)

- Given: a (0.05 or 0.01)
  - Two-tailed: z = +/- 1.96 (a = 0.05)
  - One-tailed: z = 1.64 (a = 0.05)
- Sample: r_{xy} and *n*

Hypothesis

- Null Hypothesis:
  - ***H_0: ρ = a***
- Alternative Hypothesis:
  - *H_a: ρ ≠ a* ***or***
  - *H_a: ρ ≠ 0.70* ***or***
  - ***H_a: ρ > 0.70*** *or*

Example: A sample with ***n* = 10** (x and y pairs) produced a correlation coefficient of **r_{xy} = 0.91**. Is the population correlation, *p* > 0.70?

Rejection Criteria

- We use hypothesized ρ = *0.70*
  - Underlying standard normal (*Fisher Z*)
  - Critical Value, CV:

    for one-tailed:

    $z_{0.95}$ = ***1.64***
  - Reject null hypothesis if *z-stat* >= *1.64*

Fisher Z Transformation

- Convert r to Fisher Z
  - $Z_r = 0.867$ $(r = 0.91)$
  - $Z_p = 1.528$ $(r = 0.70)$

Convert r to Fisher Z			**Calculated**
Enter r	0.70	Fisher Z is	0.86730053
Enter r	0.91	Fisher Z is	1.52752443

Standard Error

$$S_{zr} = \frac{1}{\sqrt{n-3}} = \frac{1}{\sqrt{10-3}} = 0.378$$

Test Statistics

$$z = \frac{Z_r - Z_\rho}{S_{zr}} = \frac{1.528 - 0.867}{0.378} = 1.75$$

Decision: **Approach 1**

- Critical Value
  - Given: a = 0.05, (Upper tail) z_{cv} = 1.64
- Decision: (reject H_0):
  - Since $z_{stat} > z_{cv}$ or 1.75 > 1.64
- Conclusion:
  - The $\rho > 0.70$

Approach 2: Confidence Interval

$$CI_{95} : Z_r \pm 1.96(\sigma_{zr}) = 1.75 \pm 1.96(0.378)$$

CI_{95} for Z: 1.009 to 2.491

CI_{95} for r: 0.765 to 0.986

Convert from z' to r			**Calculated**
Enter z'	1.009	Correlation, r is	0.76534809
Enter z'	2.491	Correlation, r is	0.98637283

Decision: Approach 2

- 95% Confidence Interval:
  - CI_{95} : 0.765 to 0.986
- Decision: (reject H_0):
  - Since $\rho = 0.70$ is outside CI_{95}
- Conclusion:
  - The ρ is different from 0.70