Examples: One sample case for the Correlation

Example 1. A researcher would like to know if the sample correlation coefficient obtained from analysis of 18 pairs (18 students) of data for Verbal and Quantitative scores on two standardized tests is the same as the population parameter. In former years, the researcher found that the population of students taking both tests had a correlation coefficient of, $\rho = 0.872$. The researcher obtained a $r = 0.894$ from the comparison of scores for the 18 students using SPSS. Is the sample correlation different from the population's?

Table 1. Sample Data for 18 students' Verbal and Quantitative Scores

Sample	Verbal	Quantitative
1	108	111
2	133	132
3	109	114
4	118	110
5	94	98
6	111	103
7	107	116
8	125	130
9	120	122
10	119	126
11	127	122
12	109	118
13	90	95
14	95	103
15	112	118
16	100	94
17	93	90
18	107	111

Table 2. SPSS Correlation Output for Verbal and Quantitative Scores

		quant	verbal
quant	Pearson Correlation	1	.894(**)
	Sig. (2-tailed)		.000
	N	18	18
verbal	Pearson Correlation	.894(**)	1
	Sig. (2-tailed)	.000	
	N	18	18

** Correlation is significant at the 0.01 level (2-tailed).
Note. See Figure 1 in Appendix for SPSS Correlation Procedure

Examples: One sample case for the Correlation

Null hypothesis statements:

$H_0 : \rho = 0.872$
$H_a : \rho \neq 0.872$

Significance level: $\alpha = 0.05$

Compute test statistics:

(Fisher's Z, Z_r, transformation - approximately standard normal, so we can use the z-score for our critical value for alpha = 0.05)

Standard Error of Z_r: $S_{zr} = \frac{1}{\sqrt{n-3}} = \frac{1}{\sqrt{18-3}} = \frac{1}{\sqrt{15}} = 0.258$

Test Statistics: $Z = \frac{Z_r - Z_\rho}{S_{zr}}$, where Z_r = Fisher's Z for sample r = 0.894,
Z_ρ = Fisher's Z for population ρ, and S_{zr} = standard error of Z_r .

Using "Fisher Z transformation of r Table" in Appendix or Excel Calculator:
Z_r = 1.442 (r = 0.894) and Z_ρ = 1.341 (ρ = 0.872)

Convert r to Fisher Z			**Calculatec**
Enter r	0.872	Fisher Z is	1.341366
Enter r	0.894	Fisher Z is	1.441504

$$Z = \frac{Z_r - Z_\rho}{S_{zr}} = \frac{1.442 - 1.341}{0.258} = 0.391$$

Construct Confidence Interval, CI:

CI: = Statistics ± (Critical Value)(Standard Error)
The Critical value for two-tailed standard normal z (α = 0.05) = 1.96

CI_{95} for $Z_r = Z_r \pm 1.96\ (S_{zr})$ = 1.442 ± 1.96 (0.258)
= 0.936 to 1.948 (Fisher's Z - must convert to r)

Using *Fisher Z transformation of r Table* in Appendix or Excel Calculator

The CI_{95} for r = 0.733 to 0.960

Convert from z' to r			**Calculated**
Enter z'	0.936	Correlation, r is	0.73337905
Enter z'	1.948	Correlation, r is	0.96016352

Make decision:

Criterion 1: **Confidence Interval**

Do **not** reject the null hypothesis
since population $\rho = 0.872$ is within CI_{95} for r: 0.733 to 0.960

or

Criterion 2: **Critical Value**

Do **not** reject the null hypothesis
since our test statistics, Z = 0.391 < 1.96 (critical value for $\alpha = 0.05$)

Conclusion

The sample correlation coefficient of r = 0.894 between verbal and quantitative scores is not statistically significant from the population correlation coefficient (not different from the population correlation coefficient of $\rho = 0.872$).

Example 2. A researcher would like to know if the sample correlation coefficient obtained from analysis of 18 pairs (18 students) of data for Verbal and Quantitative scores on two standardized tests is significantly different from 0. The researcher obtained a $r = 0.894$ from the comparison of scores for the 18 students using SPSS. Is the sample correlation of r = 0.894 different from an hypothesized value of 0?
(this statement is similar to asking, is there a significant correlation?)

Null hypothesis statements:

$H_0 : \rho = 0$
$H_a : \rho \neq 0$

Significance level: $\alpha = 0.05$

Compute test statistics:

(When the hypothesized population correlation is 0, the underlying distribution is the **t distribution** with $df = n - 2$ and as n get large, the sampling distribution approximates the standard normal distribution and so can use the normal z-score as the critical value)

So the critical value for df = 18 - 2 = 15 and alpha = 0.05 for two-tailed t-test is 2.12 (obtained from two-tailed t- distribution table, $t_{cv} = 2.12$)

Standard Error of Z_r: $S_{zr} = \frac{1}{\sqrt{n-3}} = \frac{1}{\sqrt{18-3}} = \frac{1}{\sqrt{15}} = 0.258$

Test Statistics: $t = \sqrt{\frac{n-2}{1-r^2}}$, where r = sample correlation coefficient of 0.894

(Test statistics for when $\rho = 0$)

$$t = r\sqrt{\frac{n-2}{1-r^2}} = 0.894\sqrt{\frac{18-2}{1-0.894^2}} = 0.894\sqrt{\frac{16}{0.201}} = 7.976$$

The p-value associated with this t-statistics is p-value = 0.000
MS Excel: ***TDIST(7.976, 16, 2)*** = > 0.000
Notice that this is same for "Sig. (2-tailed)" in Table 2

Examples: One sample case for the Correlation

Construct Confidence Interval, CI or Rejection Criterion (any one will give same result):

Approach I: Correlation Critical Value Table: (**Recommended**)

Use *Critical Values for Correlation Coefficient*, *r* in Appendix

We get r_{cv} = 0.468 (df = 18 - 2 = 16 and $\alpha = 0.05$)

Decision: since sample $r = 0.894 > 0.468$, we **reject** the null hypothesis

Approach II: Critical Value: t_{cv} (df = 16, $\alpha = 0.05$, two-tailed)

Test statistics is $t = 7.976$ and critical t or $t_{cv} = 2.12$

Decision: since t statistics > $t_{cv} = 2.12$, we **reject** the null hypothesis

Approach III: *p*-value

The *p*-value associated with this t-statistics is *p*-value = 0.000
MS Excel: ***TDIST(7.976, 16, 2)*** => 0.000
Notice that this is same for "Sig. (2-tailed)" in Table 2

Decision: since *p*-value = 0.000 < 0.05 ($\alpha = 0.05$), we **reject** the null hypothesis

Approach IV: Confidence Interval, CI_{95}

CI: = Statistics ± (Critical Value)(Standard Error)
The Critical value for two-tailed standard normal *z* ($\alpha = 0.05$) = 1.96

CI_{95} for $Z_r = Z_r \pm 1.96\ (S_{zr}) = 1.442 \pm 1.96\ (0.258)$
= 0.936 to 1.948 (Fisher's Z - must convert to r)

Using "Fisher Z transformation of r Table" in Appendix or Excel Calculator

So CI_{95} for r = 0.733 to 0.960

Decision: since $\rho = 0$ is outside the CI_{95}, we **reject** the null hypothesis

Make decision:

Reject the null hypothesis because of any of the following Criteria:

1. Critical Value for r: since sample $r = 0.894 > 0.468$ (r_{cv}) **or**
2. Critical Value for t_{cv}: since t statistics = $7.976 > t_{cv} = 2.12$ **or**
3. p-value: since p-value = $0.000 < 0.05$ ($\alpha = 0.05$) **or**
4. Confidence Interval, CI_{95}: since $\rho = 0$ is outside the CI_{95} of 0.733 to 0.960

Conclusion

The sample correlation coefficient of $r = 0.894$ between verbal and quantitative scores is statistically different from 0.

Examples: One sample case for the Correlation

Appendix

Formulas: Where z' is Fisher's Z and r is correlation coefficient

Convert from r to Fisher's Z, z': $z' = \frac{1}{2}\ln(\frac{1+r}{1-r})$

Convert from Fisher's Z to *r* : $r = \frac{e^{2z'}-1}{e^{2z'}+1}$

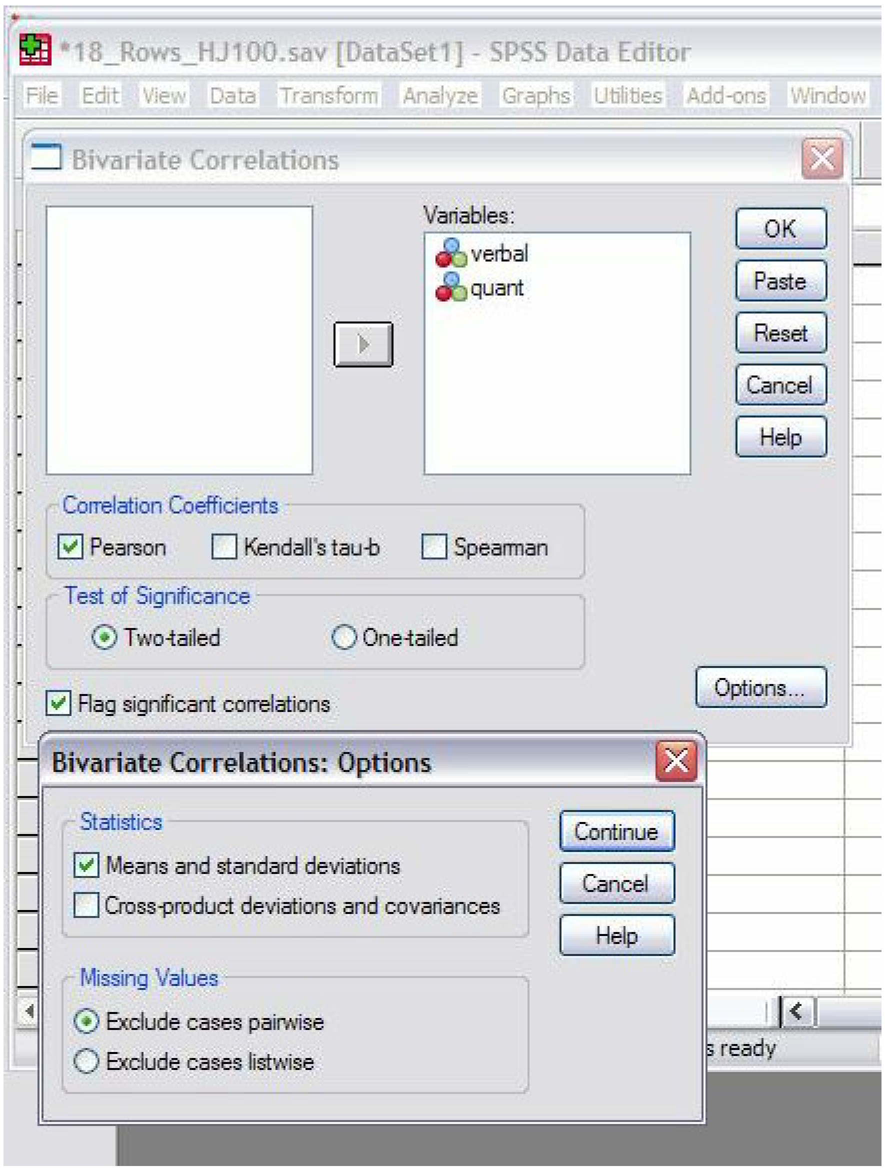

Figure 1. SPSS Correlation Procedure: **Analyze -> Correlate -> Bivarate**

Examples: One sample case for the Correlation

Fisher Z transformation of r Table

r	Fisher's Z, z'		r	Fisher's Z, z'		r	Fisher's Z, z'	
0.0000	0.0000		0.4800	0.5230		0.9500	1.8318	
0.0100	0.0100		0.4900	0.5361		0.9600	1.9459	
0.0200	0.0200		0.5000	0.5493		0.9700	2.0923	
0.0300	0.0300		0.5100	0.5627		0.9800	2.2976	
0.0400	0.0400		0.5200	0.5763		0.9900	2.6467	
0.0500	0.0500		0.5300	0.5901				
0.0600	0.0601		0.5400	0.6042				
0.0700	0.0701		0.5500	0.6184				
0.0800	0.0802		0.5600	0.6328				
0.0900	0.0902		0.5700	0.6475				
0.1000	0.1003		0.5800	0.6625				
0.1100	0.1104		0.5900	0.6777				
0.1200	0.1206		0.6000	0.6931				
0.1300	0.1307		0.6100	0.7089				
0.1400	0.1409		0.6200	0.7250				
0.1500	0.1511		0.6300	0.7414				
0.1600	0.1614		0.6400	0.7582				
0.1700	0.1717		0.6500	0.7753				
0.1800	0.1820		0.6600	0.7928				
0.1900	0.1923		0.6700	0.8107				
0.2000	0.2027		0.6800	0.8291				
0.2100	0.2132		0.6900	0.8480				
0.2200	0.2237		0.7000	0.8673				
0.2300	0.2342		0.7100	0.8872				
0.2400	0.2448		0.7200	0.9076				
0.2500	0.2554		0.7300	0.9287				
0.2600	0.2661		0.7400	0.9505				
0.2700	0.2769		0.7500	0.9730				
0.2800	0.2877		0.7600	0.9962				
0.2900	0.2986		0.7700	1.0203				
0.3000	0.3095		0.7800	1.0454				
0.3100	0.3205		0.7900	1.0714				
0.3200	0.3316		0.8000	1.0986				
0.3300	0.3428		0.8100	1.1270				
0.3400	0.3541		0.8200	1.1568				
0.3500	0.3654		0.8300	1.1881				
0.3600	0.3769		0.8400	1.2212				
0.3700	0.3884		0.8500	1.2562				
0.3800	0.4001		0.8600	1.2933				
0.3900	0.4118		0.8700	1.3331				
0.4000	0.4236		0.8800	1.3758				
0.4100	0.4356		0.8900	1.4219				
0.4200	0.4477		0.9000	1.4722				
0.4300	0.4599		0.9100	1.5275				

Examples: One sample case for the Correlation

Critical Values for Correlation Coefficient, r

Level of Significance (p) for a Two-Tailed Test				
df (n-2):	**0.1**	**0.05**	**0.02**	**0.01**
1	0.988	0.997	0.9995	0.9999
2	0.9	0.95	0.98	0.99
3	0.805	0.878	0.934	0.959
4	0.729	0.811	0.882	0.917
5	0.669	0.754	0.833	0.874
6	0.622	0.707	0.789	0.834
7	0.582	0.666	0.75	0.798
8	0.549	0.632	0.716	0.765
9	0.521	0.602	0.685	0.735
10	0.497	0.576	0.658	0.708
11	0.476	0.553	0.634	0.684
12	0.458	0.532	0.612	0.661
13	0.441	0.514	0.592	0.641
14	0.426	0.497	0.574	0.623
15	0.412	0.482	0.558	0.606
16	0.4	0.468	0.542	0.59
17	0.389	0.456	0.528	0.575
18	0.378	0.444	0.516	0.561
19	0.369	0.433	0.503	0.549
20	0.36	0.423	0.492	0.537
21	0.352	0.413	0.482	0.526
22	0.344	0.404	0.472	0.515
23	0.337	0.396	0.462	0.505
24	0.33	0.388	0.453	0.496
25	0.323	0.381	0.445	0.487
26	0.317	0.374	0.437	0.479
27	0.311	0.367	0.43	0.471
28	0.306	0.361	0.423	0.463
29	0.301	0.355	0.416	0.456
30	0.296	0.349	0.409	0.449
35	0.275	0.325	0.381	0.418
40	0.257	0.304	0.358	0.393
45	0.243	0.288	0.338	0.372
50	0.231	0.273	0.322	0.354
60	0.211	0.25	0.295	0.325
70	0.195	0.232	0.274	0.303
80	0.183	0.217	0.256	0.283
90	0.173	0.205	0.242	0.267
100	0.164	0.195	0.23	0.254

Examples: One sample case for the Correlation

Critical Values for Correlation Coefficient, r *cont.*

Level of Significance (p) for a Two-Tailed Test			
df (n-2):	**0.1**	**0.05**	**0.01**
100	0.164	0.195	0.254
120	0.151	0.179	0.234
140	0.14	0.166	0.217
160	0.13	0.155	0.203
180	0.123	0.146	0.192
200	0.117	0.139	0.182
300	0.095	0.113	0.149
400	0.082	0.098	0.129
500	0.074	0.088	0.115